*Int. J. Heat Mass Transfer.* Vol. 10, pp. 135–148. Pergamon Press 1967. Printed in Great Britain

# HEAT LOSSES THROUGH RAREFIED FLUIDS

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(*Received* 5 *May* 1966 *and in revised form* 13 *June* 1966)

**Abstract**—Heat transfers from an electrically heated tungsten filament have been investigated for a range of temperature differences, transfer fluids and test geometries at pressures ranging from $4 \times 10^{-3}$ to 770 torr. For the horizontal filament surrounded by an open-ended, coaxial cylinder $Nu$ ($\pm$ 5 per cent) = 2·35 $(Ra)^{0.12}$ in the range $10^{-4} < Ra < 1$. The limit of free molecular conduction is discussed and the onset of convection determined in terms of $Gr$ for large ratios of cylinder to filament diameters. If the axis of the cylinder is skew to the filament a minimum in the heat flux vs. pressure curve occurs in what would otherwise be the constant heat leak region. An analogy between the tested coaxial arrangement and an evacuated envelope container for liquid refrigerants is suggested.

## NOMENCLATURE

$C$, specific heat of gas [J g$^{-1}$];
$g$, acceleration due to gravity [cm s$^{-2}$];
$Gr$, Grashof number $(\Delta T . \beta . [2r_1]^3 . \rho^2 . g/\mu^2)$;
$h$, heat-transfer coefficient [Wcm$^{-2}$ °C$^{-1}$];
$k$, thermal conductivity [Wcm$^{-1}$ °C$^{-1}$];
$l$, length of filament [cm];
$m$, molecular weight of gas;
$Nu$, Nusselt number $(2hr_1/k)$;
$p$, static pressure [torr];
$Pr$, Prandtl number $(\mu . Cp/k)$;
$\dot{Q}$, rate of filament dissipation, i.e. total heat leak rate [W];
$r$, radius [cm];
$R$, universal gas constant [J °K$^{-1}$ mole$^{-1}$];
$Ra$, Rayleigh number $(Gr Pr)$;
$T$, absolute temperature [°K];
$\alpha$, surface accommodation coefficient $([T_r - T_i]/[T_s - T_i])$;
$\beta$, coefficient of volume expansion, $(1/T_g)$ [°K$^{-1}$];
$\gamma$, ratio of principal specific heats of gas $(C_p/C_v)$;
$\Delta T$, temperature difference between filament and outer tube [degC];
$\lambda$, gaseous mean free path [cm];
$\mu$, viscosity of gas [g cm$^{-1}$ s$^{-1}$];
$\rho$, density of gas [g cm$^{-3}$].

Subscripts

$c$, due to gaseous conduction;
$co$, due to gaseous convection;
$f$, molecules arriving at filament;
$g$, outer cylinder;
$i$, incident molecule;
$p$, constant pressure;
$r$, reflected molecule;
$s$, surface molecule;
$v$, constant volume;
$w$, filament;
$\infty$, due to conduction in continuum region;
1, inner cylinder;
2, external cylinder.

## INTRODUCTION

THE RADIATIVE and conductive heat transfers through gases are largely predictable. Thermal conduction in rarefied gases has been treated extensively [1, 2]: an equation has been derived by Knudsen [3] for the rate of energy loss from a heated wire in high vacua. Bomelburg [4] measured the conductive heat flux through rarefied gases surrounding very thin heated wires with particular regard to the intermediate region between the free molecular and continuum régimes. His results, and those of Schäfer *et al.* [5], have been correlated satisfactorily by the analysis (for conductive transfers between cylinders) of Lees and Liu [6].

Free convection is, in general, more difficult to analyse than either the radiative or conductive processes. Kyte *et al.* [7] extended the work of Madden and Piret [8] to derive a series of equations (applicable over a wide range of *Ra*) from which rates of convective heat loss from horizontal wires can be evaluated under conditions in which the free molecular conduction is either ignored or considered.

The sequence of experiments described in this article shows the usefulness (though rarely appreciated) of pressure as a variable, to distinguish between the convective, conductive and radiative components of heat leaks through gases (see schematic Fig. 1, the details of which will be discussed later). Such a "vacuum" analysis is often desirable as a preliminary design stage when it is necessary to minimize the total heat leak to the walls of a system (e.g. a furnace), which will in practice be used entirely at atmospheric pressure.

## EXPERIMENTAL

The apparatus (Fig. 2) consisted of a horizontal tungsten filament, 21-cm long, symmetrically placed in a coaxial, open-ended brass tube of length 15 cm, the latter being in good thermal contact with a water cooled heat sink at 16 ± 2°C. Temperature differences between the cylinder and the centre of the filament were monitored to within ±2degC by 40 s.w.g. copper–constantan and chromel–alumel thermocouples hard soldered in position. The potential differences produced were balanced to $10^{-5}$ V with a potentiometer whilst the thermoelectric circuits were floating with respect to earth. A stabilized direct current was passed through the filament.

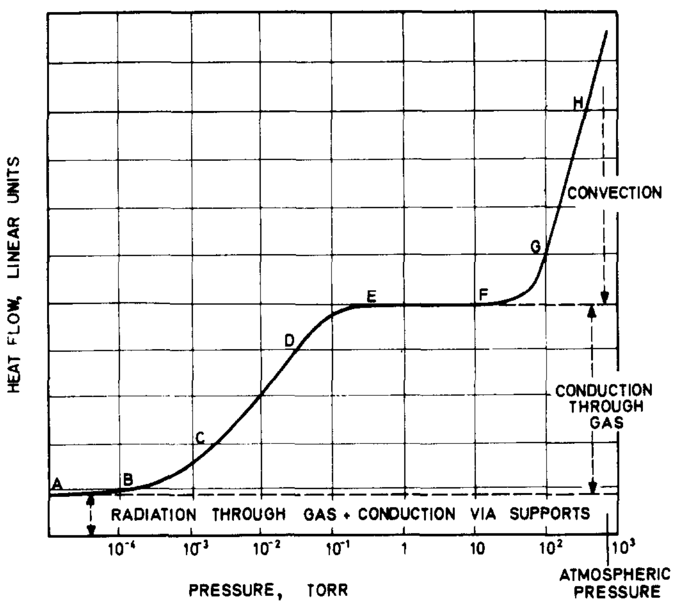


FIG. 1. Rate of energy loss from a heated (or to a refrigerated) body in an evacuated enclosure.

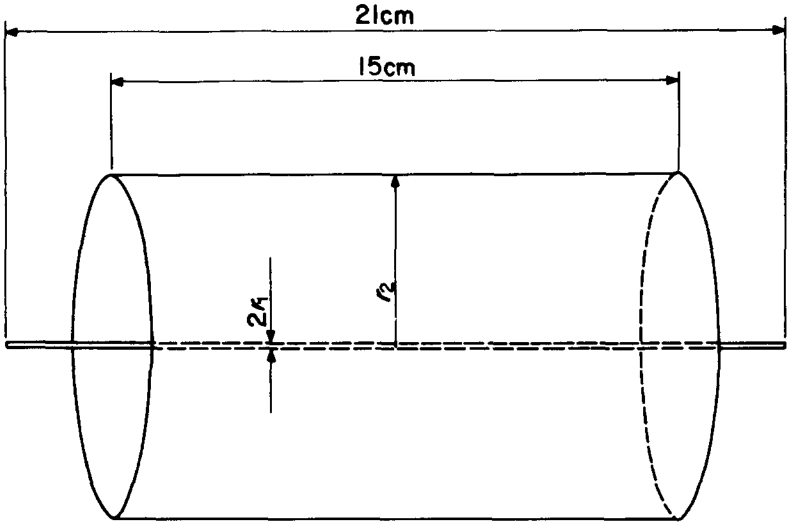


FIG. 2. Geometry of system.

the power consumed (i.e. the rate of heat generated) being determined to within 0·25 per cent from measurements of the potential drops across the filament and a standard resistor in series. The arrangement was mounted on the base of an evacuated enclosure (a glass cylindrical dome of diameter 29 cm and height 35 cm), the steady pressure during a measurement being determined with a mercury manometer in the range 760 to 5 torr and a McLeod gauge for the range 1–4 × $10^{-3}$ torr.

## CONDUCTION THROUGH RAREFIED FLUIDS

Kinetic theory indicates that the thermal conductivity of gases should be independent of pressure; in free unconfined gases this is always so because the number of molecules present increases while the mean free path decreases, both in proportion to the pressure. This independence of pressure (E F of Fig. 1) is observed in some of the experimental curves (Figs. 3–6) in the region near 10 torr. However, at higher pressures with the present geometries, convection (see F G H of Fig. 1) occurs in addition to conduction (B C D E F) and radiation through the gas and conductive heat leaks via the terminals of the filament (A B).

The temperature differences used are far greater than those with which thermal conductivity (which is a function of temperature) is measured. However, the relative values of the conductive heat transfers through helium, air and carbon dioxide, as obtained from the curves of Fig. 5, agree within experimental error with existing data when mean thermal conductivities over the appropriate temperature ranges are used.

At sufficiently low pressures the molecular mean free path $\lambda$ $\{= 8{\cdot}589\,(\mu/p)\sqrt{(T/m)}$ according to [9]$\}$ becomes greater than the dimensions of the annular gap and the heat flux is then no longer independent of pressure. This "free molecular" region is defined by $(r_1/\lambda)\,\log_e(r_2/r_1) \leqslant (1/10)$; for the continuum régime, in which the rate of heat leak is independent of pressure, $(r_1/\lambda)\,\log_e(r_2/r_1) \geqslant 75$; while between these, there is the so called transition region [6]. The present measurements corroborate the above criteria, although the condition for the free molecular domain appears to be well on the safe side.

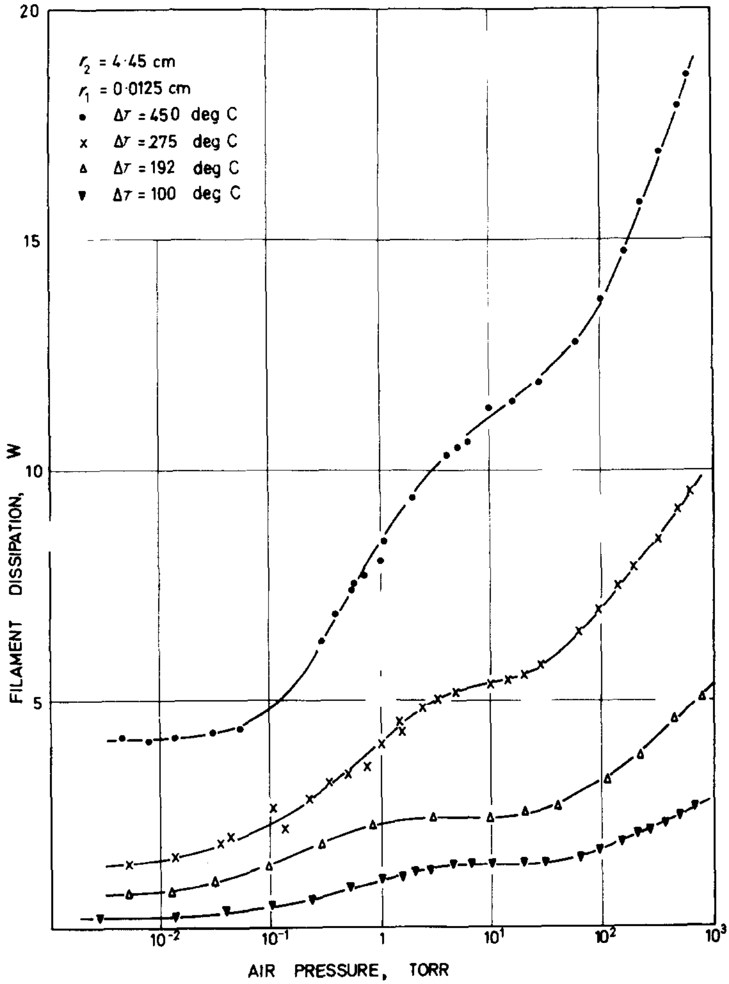


FIG. 3. Influence of filament temperature.

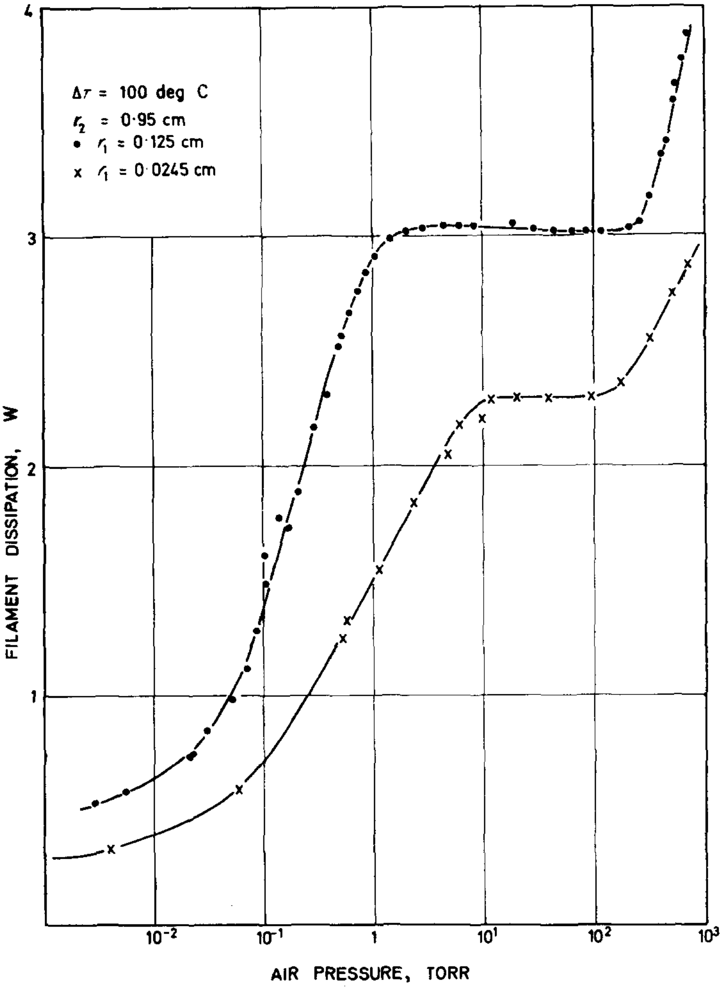


FIG. 4. Influence of filament radius.

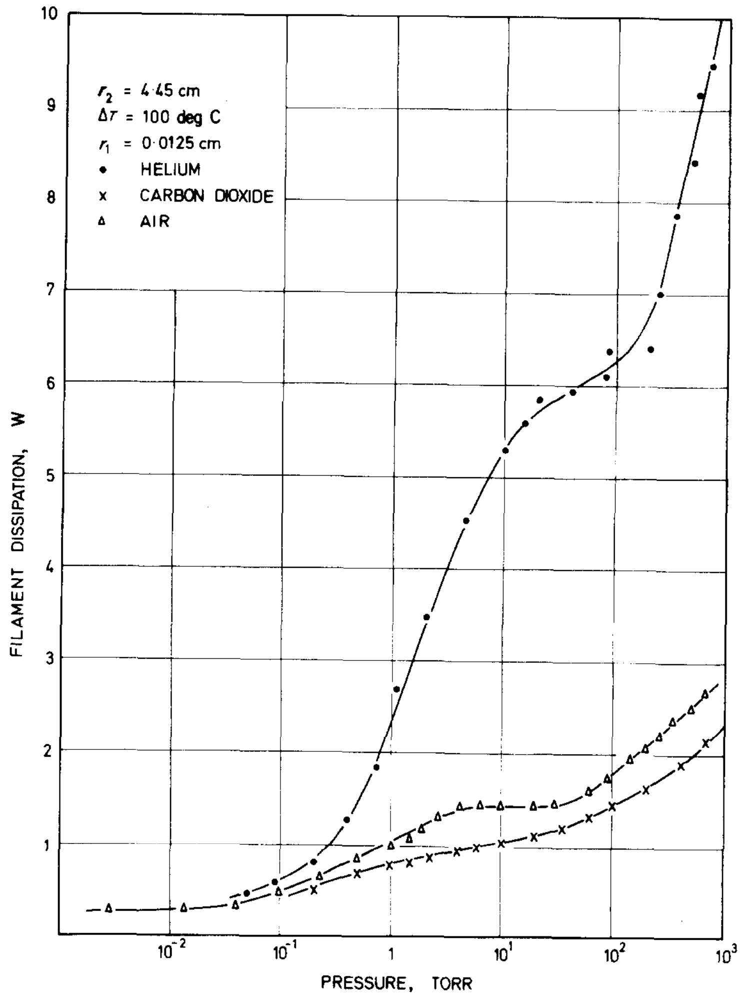


FIG. 5. Influence of transfer fluid.

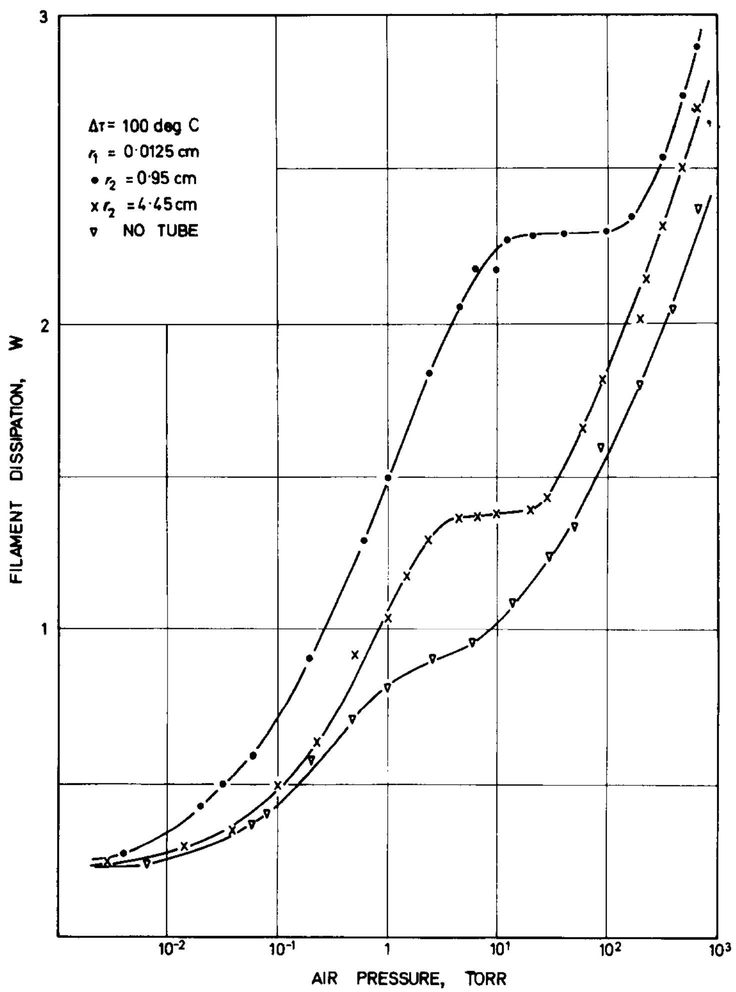


FIG. 6. Influence of tube size.

Lees and Liu [6] assumed complete thermal accommodation of the gas molecules at the surface whilst Hurlbut [10] extended the analysis to include arbitrary values for $\alpha$. Unfortunately, the available data are unsystematic [11] and, therefore, in the above criteria $\alpha$ is assumed to be unity.

Knudsen [3, 12] has deduced, for the free molecular region, that the rate of energy transfer through a gas in an annulus with large values of $(r_2/r_1)$ varies linearly with pressure and is given by

$$\dot{Q}_c = \frac{A_1}{2}\left[\frac{\alpha_1\alpha_2}{\alpha_2 + (r_1/r_2)(1-\alpha_1)\alpha_2}\right]\left\{\frac{\gamma+1}{\gamma-1}\right\}\left\{\frac{R}{2\pi T_g m}\right\}^{\frac{1}{2}}(T_w - T_f)P_g \quad (1)$$

where $A_1$ is the area of the inner cylinder. However, in practice, the situation may be more complicated—see Appendix I.

The values of $\dot{Q}_c/\dot{Q}_\infty$ determined in this study (see Fig. 7) agree well throughout the transition region (as do those of [4, 5]) with the theoretical ratios obtained from [6], and shown as continuous lines in Fig. 7, namely

$$\frac{\dot{Q}_c}{\dot{Q}_\infty} = \left[1 + \left\{\frac{4}{15}\cdot\frac{r_1}{\lambda}\cdot\log_e\frac{r_2}{r_1}\right\}^{-1}\right]^{-1}. \quad (2)$$

The agreement in the present case extends well into the free molecular region.

At low pressures, however, when $\lambda \gg r_1$, radiation and end losses become dominant, making estimations of the purely conductive component more difficult. With this effect and the assumption that $\alpha$ is unity, some deviation between the actual and theoretical ratios of $\dot{Q}_c/\dot{Q}_\infty$, especially at lower values of $r_1/\lambda$, is understandable.

## MISALIGNMENT OF FILAMENT AND CYLINDER

Figure 8 shows the results of a 5° tilting of the outer member of the coaxial system with respect to the horizontal filament. For identical boundary temperatures in both misaligned cases, the

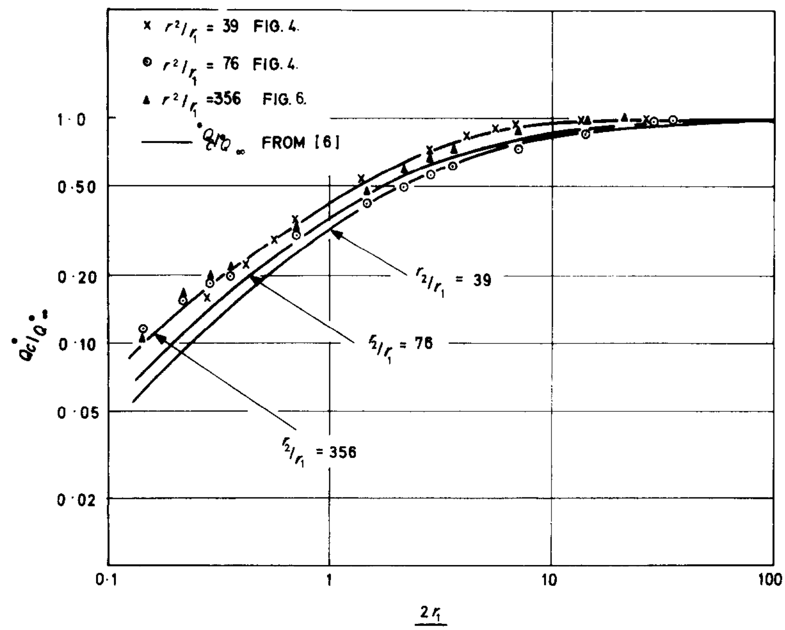


FIG. 7. Comparison of Lees and Liu theoretical curves [6] with experimental data (individual points).

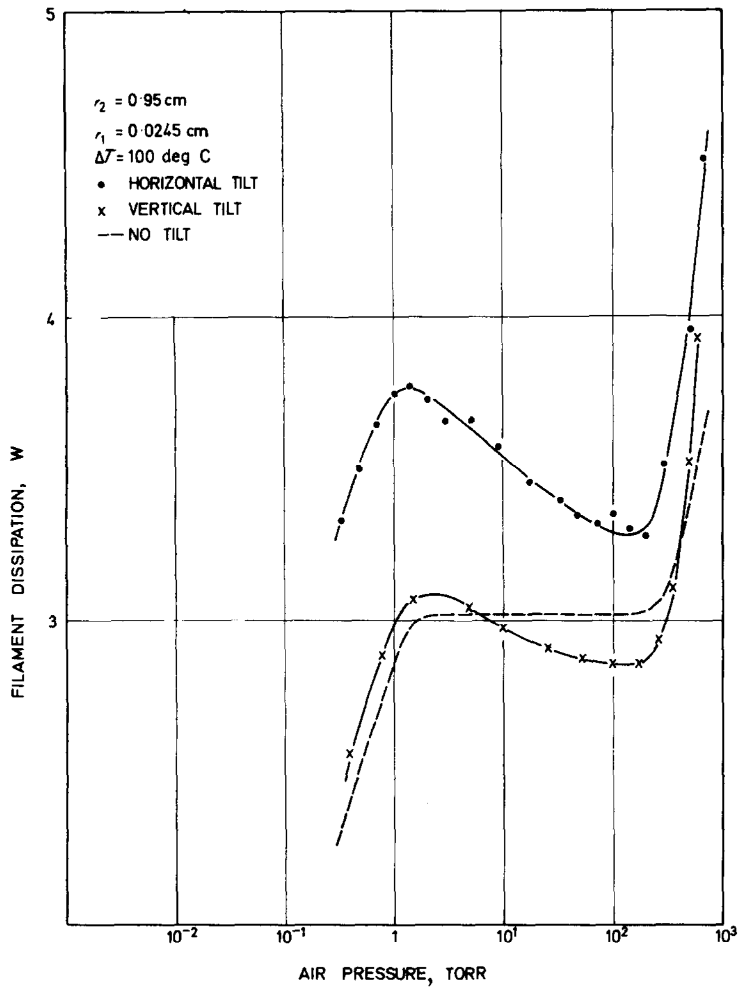


FIG. 8. Influence of relative misalignment of surrounding tube and horizontal filament.

heat leaks are greater than for the aligned arrangement. This is apparently not so for the vertically tilted case of Fig. 8, because the increased thermal contact resistance between the cylinder and the base plate resulted in the cylinder being at a higher temperature than for the undisturbed arrangement. Consequently, the temperature gradient between the cylinder and filament, and hence the heat flux, was reduced by between −5 and − 13 per cent.

At pressures above about 2 torr (in Fig. 8), normally the continuum conduction region, the heat leaks decrease to a minimum at approximately 150 torr thereafter increasing with the onset of convection. A conduction–convection interference phenomenon has also been observed by Diamant [13] and Robinson *et al.* [14], who measured the dependence of the thermal insulations of double glazing (at atmospheric pressure) upon inclination to the horizontal and air space gap. Such a minimum phenomenon, which is dependent upon geometry, temperature difference and fluid properties, has not as yet been fully exploited for the design of insulating enclosures at atmospheric pressure.

## NON-DIMENSIONAL CORRELATION OF CONDUCTION AND CONVECTION DATA

Heat leaks due to radiation and conduction via the electric terminals were subtracted from the total rates of power dissipation, so that only convection and conduction through the gas were considered in the *Nu* calculations. The non-dimensional plot is shown in Fig. 9 together with the Senftleben [15] and McAdams [16] curves for free convection from heated cylinders, but in a relatively free environment.

The results conform within experimental limits ($\pm 5$ per cent) to a curve of the form $Nu = 2{\cdot}35\,(Ra)^{0{\cdot}12}$ for the range $10^{-4} < Ra < 1$. However, it should be realized that a separate curve for each value of $r_2/r_1$ should be drawn. Fishenden and Saunders [17] obtained a family of parallel curves, each corresponding to a different value of $r_2/r_1$ from which they deduced for decreasing values of $r_2/r_1$ that *Nu* (i) decreases at high *Ra* and (ii) increases at low *Ra*. From the data of Fig. 9, the latter conclusion is verified, although contrary to the results given by Liu *et al.* [18], whose experiments covered the range $1{\cdot}15 \leqslant r_2/r_1 \leqslant 7{\cdot}5$, whilst the radius ratios in the present study varied between 39 (Fig. 4) and 356 (Fig. 6). In all cases the effective *Nu* was greater than that predicted by the McAdams curve [16]: this is in accordance with the suggestion of Senftleben and Gladisch [19] that internal flow effects remain important for $r_2/r_1$ ratios of up to 500.

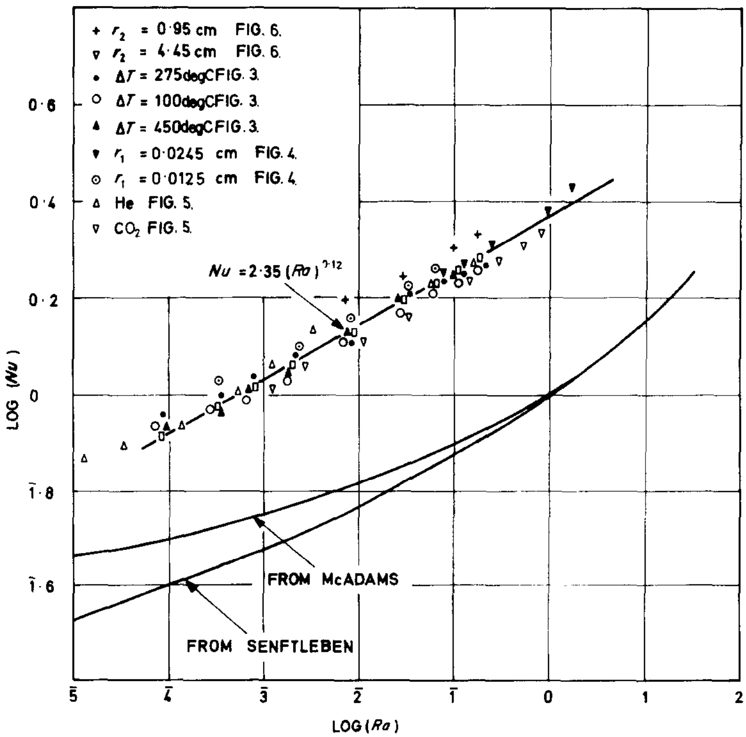


FIG. 9. Non-dimensional plot of the conductive and convective data.

## CONVECTION

Various values of *Gr* using the annular gap for the characteristic length have been suggested as a limit below which convection is negligible for a cylindrical system: that generally accepted

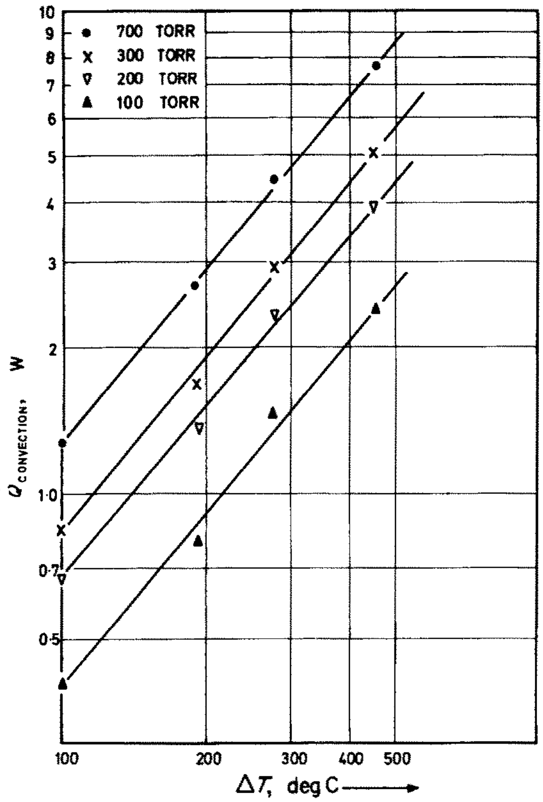


FIG. 10. Convective heat flux dependence upon temperature difference between filament and surrounding tube.

for small $r_2/r_1$ ratios is $2{\cdot}0 \times 10^3$ [20]. The onset of convection, corresponding to F in Fig. 1, can be seen in Figs. 3–6 and the appropriate values of *Gr* based on the width of the annular gap as the linear dimension are, in all instances, in the region of $2{\cdot}4\ (\pm 0{\cdot}2) \times 10^3$.

It is shown in Fig. 10, which is deduced from the data of Fig. 3, that $\dot{Q}_{co}$ is proportional to $\Delta T^{1{\cdot}21 \pm 0{\cdot}06}$ for pressures greater than 100 torr.

Decreasing the cylindrical tube radius increasingly inhibits convection as seen in Fig. 11, whilst the onset of convection (see Figs. 4 and 6) occurs at higher pressures if the filament diameter is increased or the diameter of the outer tube decreased. It is interesting to note that in the convective region, $d\dot{Q}/d(\log p)$ is independent of the presence or diameter of the outer coaxial tube and increases with (i) increasing temperature difference between the filament and the tube, (ii) increasing filament diameter and (iii) decreasing molecular weight of the transfer fluid. However, the pressure sensitivity is much less than for the free molecular region (where the arrangement is known as the Pirani gauge) and it further decreases as the pressure rises. Nevertheless, this type of arrangement would be

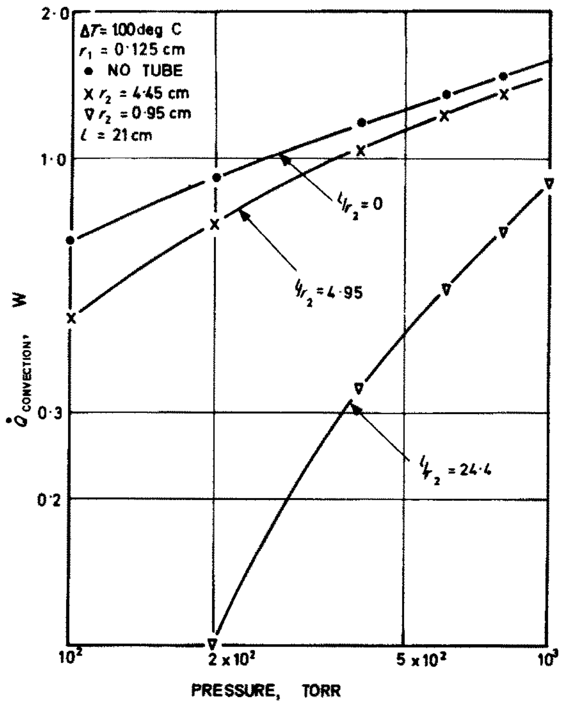


FIG. 11. Convection heat flux dependence upon static pressure (data taken from Fig. 6).

useful for leak detection in the convection region using say helium as the search gas.

**ACKNOWLEDGEMENT**

We wish to thank F. G. Williams, of the Basic Physics Department of the British Oxygen Co. Ltd. London, for discussing conductive heat leaks through cryogenic vapours with us.

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**APPENDIX I**

*Heat leaks through low-pressure vapours*

For any real gas or vapour $\gamma$ is dependent upon temperature and pressure. At 4·2°K, $\gamma$ for $He^4$ varies from 2·03 to 1·67 as the pressure is reduced from 1 atm to effectively zero [21, 22], and thus at $10^{-3}$ torr a value of 1·67 may be assumed. Hydrogen vapour at 20·4°K is in a degenerate state and behaves at low pressures as an ideal monatomic gas with $\gamma = 1{\cdot}67$. Using these data in equation (1), the heat leaks per second per unit area between two parallel plates per degC temperature difference through rarefied vapours have been calculated (see Fig. 12) for the common cryogenic fluids at their normal boiling points assuming $\alpha = 1$ for both surfaces. These heat leaks are frequently ignored when measuring the conductivity of solid thermal insulators, so leading to large errors. The situation, however, need not be as bad as the graphs (Fig. 12) imply, because with helium or hydrogen in contact with very clean surfaces (an unusual engineering situation), $\alpha$ may be as low as 0·02. Nevertheless, in practice, for rough surfaces and for the heavier gases it is safer to assume a value for $\alpha$ of near unity.

Figs. 3–6 bear a qualitative similarity to the variation with envelope pressure of heat leaks into storage vessels for cryogenic liquids [23, 24]. It is suggested that the present experimental arrangement provides a convenient analogue for investigations into the improvement of the design of such vessels.

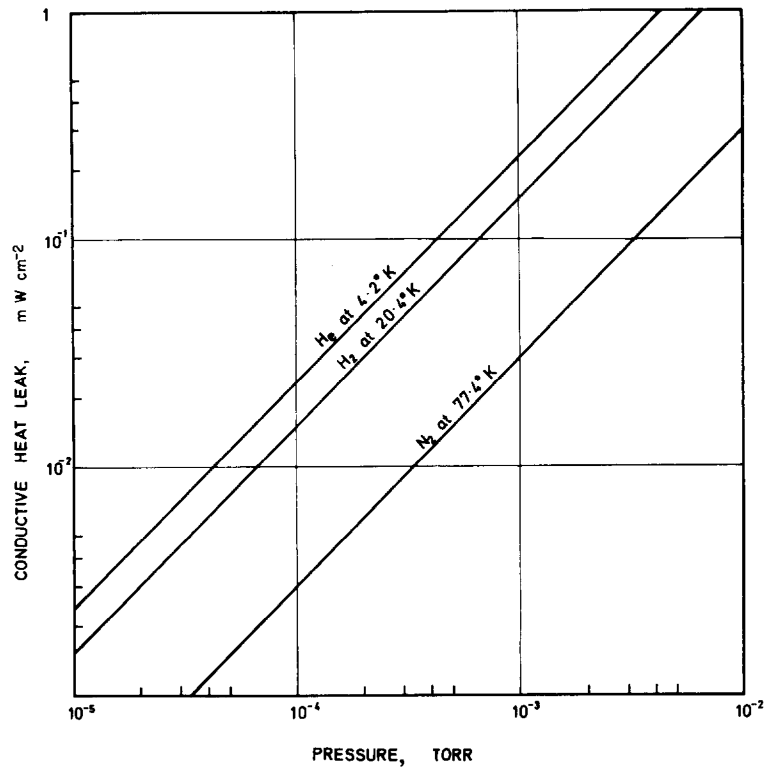


FIG. A1. Conductive heat flux per unit area through rarefied fluids at their normal boiling points.

**Résumé**—On a étudié le transport de chaleur à partir d'un filament de tungstène chauffé électriquement pour plusieurs différences de températures, fluides de transport et géométries essayées à des pressions allant de $4 \times 10^{-3}$ à 770 torr. Pour le filament horizontal entouré par un cylindre coaxial à extrémités ouvertes, Nu ($\pm 5\%$) = 2,35 $(Ra)^{0,12}$, dans la gamme $10^{-4} < Ra < 1$. La limite de la conduction en régime moléculaire libre est discutée et le début de la convection déterminé sous la forme du nombre de Grashof $Gr$ pour des rapports élevés des diamètres du cylindre et du filament. Si l'axe du cylindre est incliné sur celui du filament, un minimum de la courbe flux de chaleur en fonction de la pression se produit dans ce qui aurait été autrement la région de perte de chaleur constante. On suggère une analogie entre la configuration coaxiale essayée et une enveloppe double (dans laquelle on a fait le vide) de réservoir pour des réfrigérants liquides.

**Zusammenfassung**—Der Wärmeübergang von einem elektrisch beheizten Wolframfaden wurde untersucht für einen Bereich von Temperaturdifferenzen, Übergangsmedien und Versuchsanordnungen bei Drücken von $4 \times 10^{-3}$ bis 770 Torr. Für den waagerechten Faden der von einem koaxialen, an den Enden offenen Zylinder umgeben ist, ergibt sich $Nu$ ($\pm 5\%$) = 2,35 $(Ra)^{0,12}$ im Bereich $10^{-4} < Ra < 1$. Die Grenze der molekularen Leitfähigkeit wird diskutiert und der Beginn der Konvektion als Ausdruck von $Gr$ bestimmt für grosse Verhältnisse von Zylinder- zu Fadendurchmesser. Bei gegenseitiger Neigung der Achsen von Zylinder und Draht erscheint ein Minimum in der Kurve des Wärmeflusses über dem Druck in einem sonst konstanten Bereich der Wärmeverluste. Eine Analogie zwischen der untersuchten Koaxialanordnung und einem Behälter mit evakuierter Umhüllung für flüssige Kühlmittel wird vorgeschlagen.

**Аннотация**—Перенос тепла от нагреваемой электрическим током вольфрамовой нити накаливания изучен для ряда температурных напоров, теплоносителей и конфигураций при давлении от $4 \times 10^{-3}$ до 770 тор. Для горизонтальной нити накаливания в коаксиальном открытом с обоих концов цилиндре $Nu\ (\pm 5\%) = 2{,}35\ (Ra)^{0{,}12}$ в диапазоне $10^{-4} < Ra < 1$. Рассматривается предел свободной молекулярной проводимости и определяется возникновение конвекции в зависимости от *Gr* при больших отношениях диаметров цилиндра и нити накаливания. Если ось цилиндра расположена наклонно к нити накаливания, имеет место минимум кривой зависимости теплового потока от давления, в противном случае имеет место область постоянной утечки тепла. Проводится аналогия между исследуемым коаксиальным устройством и вакуумным контейнером с жидким охладителем.